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Topology, fermionic zero modes and flavor singlet correlators in finite temperature QCD.

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We extend our earlier results concerning the breaking of the $U_A(1)$ symmetry in finite temperature QCD. In particular, we use topological argments to investigate the chiral limit.

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Among the non-perturbative properties of quarks, those associated with the topology of the gauge fields occupy a special place. This follows from the Atiyah-Singer index theorem which guarantees the existence of fermionic zero modes on configurations with non-trivial topology. As a result, non-perturbative issues related to the axial anomaly can generally be studied with enhanced control. In the context of finite temperature QCD, one such issue relates to the realization of the anomalous $U(1)$ axial symmetry. If this symmetry were to be effectively restored at the critical temperature, the increased number of soft degrees of freedom would modify the nature of the phase transition [1]. Needless to say, only the lattice can really hope to tackle this question, at this time. Unfortunately, the lattice formulation also introduces artifacts and there is for example no exact index theorem in this context. Instead, what is observed with the actions currently used is the "zero mode shift" phenomenon [2] (i.e. modes with small but non-zero eigenvalue and which are not exactly chiral). What we would like to show here is that even in this case, enough is recognizable of the continuum behavior that meaningful conclusions can be drawn. We find that the $U_A(1)$ symmetry is not restored at the phase transition but only at somewhat higher temperatures. At the same time, it is clear that detailed quantitative questions could only be answered with the use of an improved action.

Some of the results presented here are an extension of preliminary data introduced last year [3]. Many of the trends are confirmed. In addition, we discuss the issue of reaching the chiral limit. We have measured the screening correlators in the scalar and pseudoscalar flavor singlet and flavor triplet sectors. From there, one can investigate the symmetry restorations by looking for degeneracies, either in the fitted screening masses or in the susceptibilities (integrated correlators). These computations were carried out on a set of configurations from the HTMCGC collaboration [4]. These were generated on a lattice of size $16^3 \times 8$ with 2 flavors of dynamical staggered fermions with mass $ma = 0.00625$. In addition to those, we have generated a new set of configurations at $\beta = 5.4875$ which is just above the crossover temperature at this value of the quark mass. Our results for the screening masses are presented in figure 1. The new measurements at $\beta = 5.4875$ confirm the trend observed earlier, namely that the σ becomes light close to the transition while the δ does not. It is worth mentioning here that some of the fits presented in fig.1 may have large systematic errors (only the statistical errors are included in the figure). This is due in part to the small extent of the lattice and, in the case of the σ, to the additional difficulties associated with the measurements of disconnected quark loop correlators. Nevertheless, it seems to us that the trends observed in fig. 1 are rather clear. In addition, they are consistent with our measurements of the susceptibilities (as

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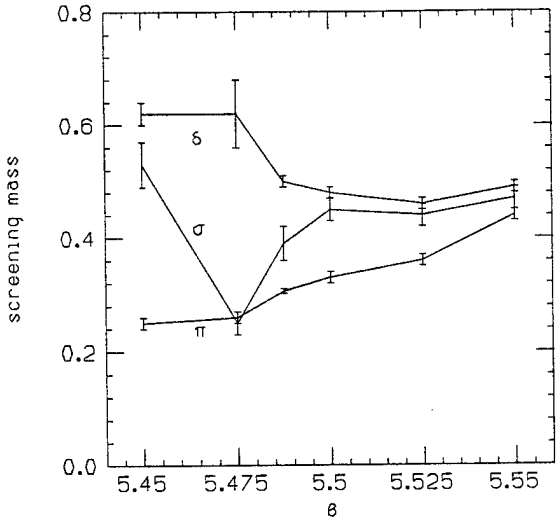

Figure 1. Screening masses in two flavor QCD

well as those by other groups [5]), in that the peak of the scalar susceptibility is seen in the disconnected and not the connected contribution (for summary plots of earlier results see ref [6]). If this situtation persists in the chiral limit, it will imply a non-restoration of the $U_A(1)$ symmetry in the high temperature phase of QCD. Namely, the π and σ become light close to the transition, but not the δ.

In the case of the susceptibilities, extrapolations to the chiral limit at fixed β have been attempted by two groups but the results appear ambiguous [7,8]. For example, it is seen that linear fits generally have a better χ^2 than fits to a quadratic curve which would be expected on theoretical grounds. Here we would like to argue that these difficulties are due to lattice artifacts and will propose a method which allows us to extract information about the chiral limit without carrying out an explicit extrapolation. The idea is to identify the reason for the non-vanishing of the disconnected susceptibility in the chiral limit. This is done by making use of the spectral decomposition of the quark propagator and the resulting formulas:

$$S(x,y) = \sum_\lambda \frac{\psi_\lambda(x)\psi_\lambda^\dagger(y)}{i\lambda+m} \quad (1)$$

$$Q \equiv TrS = \sum_{\lambda>0} \frac{2m}{\lambda^2+m^2} + \frac{n_L+n_R}{m} \quad (2)$$

$$Q_5 \equiv Tr\gamma_5 S = \frac{n_L-n_R}{m} \quad (3)$$

where n_L (n_R) are respectively the number of left (right) zero-modes of the Dirac operator. In terms of these the scalar and pseudoscalar disconnected susceptibilities are written $\chi^{dis} = [< Q^2 > -(< Q >)^2]/V$ and $\chi_5^{dis} =< Q_5^2 > /V$. Now it is clear that only the contribution from fermionic zero-modes will survive at $m = 0$ and that the chiral limit will be smooth. Remember that for $N_f = 2$ the fermionic determinant picks up a factor of m^2 for each zero mode. On configurations with topological charge one, the factors of m will cancel from measurements of $\chi_{(5)}^{dis}$ and leave a well defined non-zero result. This is what we expect in the continuum, on the lattice however things will be more complicated. First, there are no exact zero modes (except on a subspace of measure zero). Then:

$$Q^{Latt} = \sum_{\lambda>0} \frac{2m}{\lambda^2+m^2} \quad (4)$$

$$Q_5^{Latt} = \sum_{\lambda>0} \frac{2m<\psi_\lambda|\Gamma_5|\psi_\lambda>}{\lambda^2+m^2} \quad (5)$$

where we have used the symmetries of the staggered action and Γ_5 is the (four-link) lattice γ_5 operator. It is then clear that on the lattice, disconnected susceptibilities will *vanish* in the chiral limit and be proportional to m^2 for small m. We believe that it is this lattice artifact which makes a chiral extrapolation from the lattice data extremely difficult [7,8].

We therefore propose a different method for studying the $m \to 0$ limit. The idea is to identify "continuum behavior" in the lattice data (and then use what we know about the continuum theory). Together with the Atiyah-Singer index theorem, (3) implies $mQ_5 = Q_{top}$. We would like to see the same connection with topology on the lattice. The formula (3) comes about because in the continuum $< \psi_\lambda|\gamma_5|\psi_\lambda >$ is either ± 1 if $\lambda = 0$ or 0 otherwise. The lattice expression (5) will closely match the continuum if we see on the lattice a clear correlation between small eigenvalues and large $r_\lambda \equiv< \psi_\lambda|\Gamma_5|\psi_\lambda >$ (i.e. if we can identify chiral modes). Figure 2 is a plot of r_λ versus λ for the lowest 8 positive eigenvalues on each configuration of our sample at $\beta = 5.55$. Many of

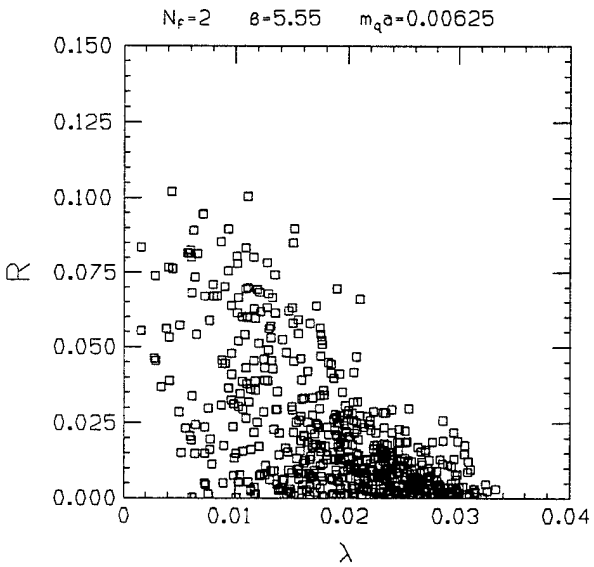

Figure 2. (r, λ) diagram at $\beta = 5.55$

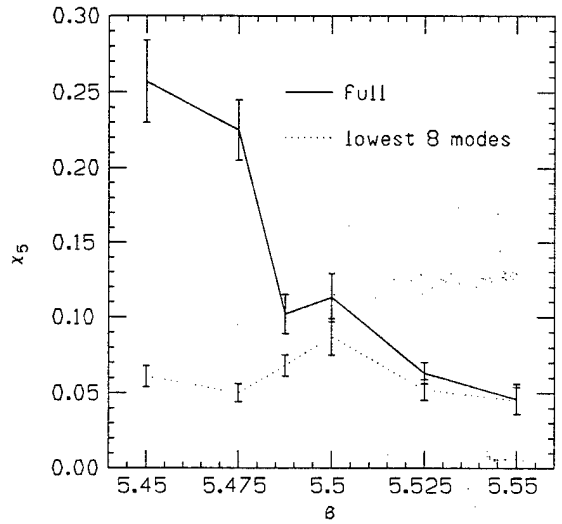

Figure 3. disconnected γ_5 susceptibility

the small eigenvalues are indeed associated with a larger value of r_λ. Of course, the quality of the correlation depends on how strong the coupling is (It is for example not as clear here as it was in quenched QCD at $\beta = 6.2$ [3]). The second step in the argument consists in showing that these low modes (the would be chiral modes of the continuum theory) indeed saturate the lattice results. In figure 3 we compare the full result for χ_5^{dis} with the one obtained by truncating (5) to the lowest 8 modes. The agreement is perfect at $\beta = 5.55$ and deteriorates somewhat in the rest of the symmetric phase (in part because of lattice artifacts). Putting everything together, we find that χ_5^{dis} is non-zero in the symmetric phase and that the topological origin of this result guarantees that it will remain non-zero in the *continuum* chiral limit. We therefore conclude that the $U_A(1)$ symmetry is not restored at the phase transition [3]. Our results also indicate the importance of investigating improved lattice actions which better satisfy the index theorem. These would allow the continuum behavior to appear even more clearly than here. Also, lower values of the quark mass would be allowed before the zero-mode shift phenomenon becomes a problem. Certainly, such actions will be necessary to obtain quantitative results. In fact, some of the formulations that one would like to consider have been mentioned at this conference (domain wall fermions, perfect action, ...). Beyond that, there are many other questions which can be investigated with the knowledge of the low lying eigenvalues and eigenvectors. Correlations between the eigenvalues is one of them. The relation with instantons and the possible existence of instanton- anti-instanton molecules is another.

[3] Of course, this conclusion only concerns the volume studied so far, namely $V = (2/T)^3$. The actual volume dependance remains to be investigated.

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