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QCD with chiral 4-fermion interactions (χQCD)*

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Lattice QCD with staggered quarks is augmented by the addition of a chiral 4-fermion interaction. The Dirac operator is now non-singular at $m_q = 0$, decreasing the computing requirements for light quark simulations by at least an order of magnitude. We present preliminary results from simulations at finite and zero temperatures for $m_q = 0$, with and without gauge fields.

1. INTRODUCTION

χQCD is QCD with a chiral 4-fermion interaction (Nambu–Jona-Lasinio—Gross–Neveu)[1,2]. Since the 4-fermion term is an irrelevant operator, this theory should lie in the same universality class as regular QCD.

On the Lattice we use staggered quarks. For simplicity we consider a theory where the 4-fermion operator has the

$$U(1) \times U(1) \subset SU(N_f) \times SU(N_f)$$

flavour symmetry generated by $(1, i\gamma_5\xi_5)$ [3]. The molecular dynamics Lagrangian for the lattice theory is

$$\begin{aligned} L &= -\beta \sum_{\square}[1 - \frac{1}{3}\mathrm{Re}(\mathrm{Tr}_{\square} UUUU)] + \sum_{s} \dot{\psi}^{\dagger} A^{\dagger} A \dot{\psi} \\ &\quad - \sum_{i} \frac{1}{8} N_f \gamma(\sigma^2 + \pi^2) + \frac{1}{2}\sum_{i}(\dot{\sigma}^2 + \dot{\pi}^2) \\ &\quad + \frac{1}{2}\sum_{l}(\dot{\theta}_7^2 + \dot{\theta}_8^2 + \dot{\theta}_1^* \dot{\theta}_1 + \dot{\theta}_2^* \dot{\theta}_2 + \dot{\theta}_3^* \dot{\theta}_3) \qquad (1) \end{aligned}$$

where

$$A = \not{D} + m_q + \frac{1}{16}\sum_{i}(\sigma_i + i\epsilon\pi_i) \qquad (2)$$

with $\epsilon = (-1)^{x+y+z+t}$. This describes 8 flavours. For N_f which is not a multiple of 8 we use "noisy" fermions [4] and multiply the fermion kinetic term by $N_f/8$.

The Dirac operator of regular lattice QCD becomes singular as $m_q \to 0$, whereas that for χQCD remains non-singular at $m_q = 0$. Conjugate gradient inversion of the regular QCD Dirac operator requires a number of iterations which $\to \infty$ as $m_q \to 0$. Inversion of χQCD Dirac operator requires a finite number of iterations even at $m_q = 0$. In practical terms, conjugate gradient inversion of the regular QCD Dirac operator requires ~10,000 iterations at the physical u and d quark masses. Simulating χQCD at $m_q = 0$, the worst we have found is ~ 400 iterations — a saving of a factor of ~ 12 in CPU time! χQCD allows us to work *at* the chiral limit, which is often useful.

All the simulations which I will now describe are being performed at $m_q = 0$. Simulations are being performed on the CRAY C-90 and CRAY J-90 at NERSC.

2. χQCD AT FINITE TEMPERATURE

We are studying the finite temperature behaviour of χQCD with $N_f = 2$ and $m_q = 0$, using the hybrid molecular dynamics algorithm with "noisy" quarks, as a function of β and γ.

We are studying the deconfinement and chiral transitions — not in general coincident at finite a. The 2 transitions come together as $a \to 0$ and in general as γ is increased. By measuring the critical behaviour of such observables as $\langle\bar{\psi}\psi\rangle$, we

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hope to determine the equation of state, where these transitions coincide. For this, being able to work *at* the chiral limit is a boon.

Our present simulations are being performed at $N_t = 4$, using $8^3 \times 4$ lattices to find the position of the phase transitions, and $12^2 \times 24 \times 4$ lattices to study the transition in detail (the extension of this lattice in the z direction is to allow the measurement of hadron screening masses).

At $\gamma = 2.5$ the deconfinement transition occurs between $\beta = 5.5$ and $\beta = 5.6$ while the chiral transition is at $\beta \sim 10$. At $\gamma = 5$ the transitions still appear separated, but clearly the chiral condensate feels the deconfinement transition. By $\gamma = 10$ the 2 transitions are close enough together that it is no longer clear whether they are distinct – see figure 2. The way we measure $\langle\bar{\psi}\psi\rangle$, it does

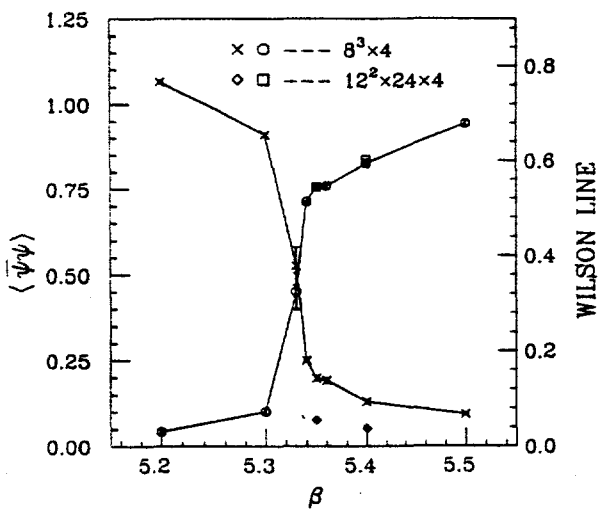

Figure 1. Wilson line and $\langle\bar{\psi}\psi\rangle$ as functions of β for $\gamma = 10$

not vanish in the symmetric phase, but should instead behave as $1/\sqrt{V}$. We have tested this at $\gamma = 10$, $\beta = 5.4$ and $\beta = 5.35$, and find it to be true within errors which indicates that both these β's lie in the chirally symmetric – deconfined – phase. (The phase transition occurs at $\beta \approx 5.33$).

3. ZERO TEMPERATURE χQCD

We are simulating χQCD with $N_f = 2$ and $m_q = 0$ at $\gamma = 10$ and $\beta = 5.4$ on an $8^3 \times 24$ lattice to study zero temperature physics and in particular hadron spectroscopy.

We have measured the propagators of the σ and π auxiliary fields on 2500 configurations separated by 2 time units. The first 500 were discarded leaving 2000 for analysis. These were binned into 200 bins of 20 time units in length for error analysis and fitting. The pion propagator was fitted to the massless scalar lattice propagator (see figure 3). The fit, over time interval 1–12, had confidence level 78% showing that it is indeed a Goldstone pion. The σ propagator is

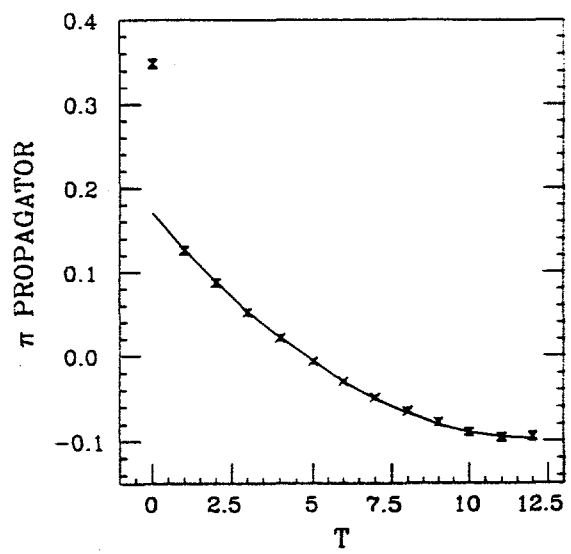

Figure 2. The propagator for the Goldstone pion. The fit is to a massless scalar lattice propagator.

noisy, as expected. The best fit we were able to obtain gave a mass of 1.12(23). Current statistics preclude determining whether there is any significant contribution from the 2-pion cut.

So far we have stored 250 configurations spaced by 20 time units for calculating the hadron spectrum. We plan first to calculate the local meson and baryon spectra. For the mesons, this will include the normal, chirality 1 mesons (quarks having chirality $\pm\frac{1}{2}$) formed from linear combinations of

$$\mathrm{Tr}[G(\pi)G^{\dagger}(-\pi)] \tag{3}$$

and the chirality 0 mesons formed from linear combinations of

$$\mathrm{Tr}[G(\pi)G^{\dagger}(\pi)] \tag{4}$$

where G is the quark propagator. For the normal meson propagators, the σ and π have disconnected contributions which we will also need to calculate, while the ρ and a_1 do not. For the chirality 0 mesons there are no disconnected contributions.

Working at $m_q = 0$ we will need a careful finite size analysis of the hadron spectrum, since hadron sizes are determined by m_π.

4. χQCD AT ZERO GAUGE COUPLING

The $\beta \to \infty$ limit of χQCD with $N_f = 2$ is the 4-d Nambu–Jona-Lasinio—Gross–Neveu model with $N_f = 6$. We have studied this theory on an 8^4 lattice to find its chiral transition, which enables us to identify its strong and weak coupling domains. We find the transition γ to be ~ 1.7. We have also studied its finite temperature behaviour on an $8^3 \times 4$ lattice, where the transition occurs at $\gamma \sim 1.5$.

5. COMMENTS ON χQCD AT FINITE BARYON NUMBER DENSITY

Quenched lattice QCD has problems at finite baryon number density. If μ is the quark number chemical potential, one would expect QCD to have a phase transition at $\mu \approx m_N/3$. Instead, at $m_q = 0$, quenched QCD appears to have a transition at $\mu = 0$ [5].

The nearest thing to a quenched version of massless χQCD we know is quenched QCD with fermions interacting with a mean field approximation to the chiral fields in which σ is replaced with a constant and the π with zero. This is just massive quenched QCD ($mass =< \sigma >$) at finite chemical potential, which has a transition at $\mu \approx m_\pi(< \sigma >)/2$ which is no longer zero. Since we expect that $m_\pi(< \sigma >)/2 < m_N(< \sigma >)/3$ we have not completely solved the problem. However, for moderate $< \sigma >$, the behaviour is more physical than for conventional quenched QCD — the unphysical region from $m_\pi(< \sigma >)/2$ to $m_N(< \sigma >)/3$ is small.

6. SUMMARY AND CONCLUSIONS

χQCD enables simulations at physical u and d quark masses with at least an order of magnitude saving in CPU time. It also enables simulations with zero quark masses which is important for determining the equation of state. A renormalization group analysis will be needed to continue to the continuum limit.

After finishing this first round of simulations and hadron mass/screening mass measurements at zero and finite temperature, we will move to larger lattices, and also perform some simulations at non-zero quark mass. The next stage will involve using a more physical chiral group ($SU(2) \times SU(2)$??).

Auxiliary fields and constituent quark masses give promise of an effective Lagrangian interpretation of χQCD. These effective quark masses could (partially) resolve the problems at finite baryon number density.

Similar theories have been considered by Brower, Orginos, Shen and Tan [6]. Theories with 4-fermion and gauge interactions have a long history; see for example [7].

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